

Problem for the week of February 1, 2010

Let A be an $n \times n$ real symmetric positive definite matrix. If B is an $n \times m$ real matrix, show that $B^T A B$ is positive semidefinite. Also show that $\text{rank}(B^T A B) = \text{rank} B$, so that $B^T A B$ is positive definite if and only if B has rank m .